What It Is Like on the Corners:
Gravity, Geodesy, Climate and Bearing Strength at the Vertices of the Lunar Cube

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Abstract

Three papers of this series have established that the Moon is a cube of edge $a_\mathrm{M} = 2800.7$ km (*HER*), that its cubic form requires a curved Earth (*EvHR*), and that its apparent roundness is produced one twentieth by optics and nineteen twentieths by the visual cortex (*FAHR*). All three concerned the cube as seen. The present paper concerns the cube as *stood upon*, and in particular its eight vertices, which every previous paper has invoked and none has visited. Using the closed-form Newtonian potential of a homogeneous rectangular prism (MacMillan 1930; Waldvogel 1976) we compute the gravity field of the lunar cube exactly. At the centre of a face $g = 1.6223$ m s^{-2}, within 0.06% of the spherical value 1.6233 m s^{-2}, which is why no Apollo gravimeter noticed; at an edge midpoint $g = 1.371$ m s^{-2} and the local vertical is tilted 45° from the face normal; at a vertex $g = 1.049$ m s^{-2} and the tilt is $\arctan\sqrt{2} = 54.74°$. The face-centre equipotential lies 394 km below the edges and 650 km below the vertices, so that on the cube "downhill" means "towards the middle of the face" everywhere, and regolith, which *HER* showed to be Banach–Tarski-rearranged and hence mobile, pools on the faces to form the maria and leaves the vertices as bare highlands, in agreement with the selenographic record. We compute the surface geometry at a vertex: the Gaussian curvature is a Dirac mass of $\pi/2$ (Descartes), geodesics through a vertex are non-unique, and a surveyor who walks a closed loop around one returns with her compass rotated by exactly 90° (holonomy equals angular defect), a result we name the Corner Compass Anomaly and propose as the definitive field test of *HER*. The thermal regime of a Lambertian face is shown to be spatially uniform with a discontinuous terminator at the edges, and the vertex to be the only place on the Moon from which the Sun can be seen for three quarters of the lunar day. Finally we address the bearing problem: a vertex of 650 km geopotential height exerts a basal pressure of 3.7 GPa, ten times the strength of rock. We resolve it with the observation that the lunar core, being a Banach–Tarski assembly of non-measurable pieces (Vitali 1905; Hausdorff 1914), has no Lebesgue volume and therefore no weight, so that the corners are held up by nothing and require nothing to hold them. What it is like on the corners is, in a word, uphill.

Keywords: lunar cube; vertex; potential of a homogeneous cube; local vertical; equipotential; holonomy; angular defect; Corner Compass Anomaly; non-measurable core; Kepler's *Somnium*.

1 Introduction

HER [1] proved that the Moon has eight corners. *EvHR* [2] used them as geodetic marks. *FAHR* [3] explained why nobody sees them. At the end of *FAHR* the referee asked the obvious question—what is it like on them?—and the authors, in the manner of the Society, promised a paper. This is it.

The title is borrowed from Nagel [13], who asked what it is like to be a bat and concluded that a physical description, however complete, does not answer the question. We respectfully disagree in the present case. A vertex of a homogeneous cube is a place with a definite gravitational acceleration, a definite local vertical, a definite temperature, a definite view and a definite curvature, all of which can be computed, and we shall compute them. What remains of Nagel's question after Sections 2–6 is, we submit, small, and we address it in Section 8.

We stress continuity with the earlier papers. The cube is homogeneous with the lunar mass $M_\mathrm{M} = 7.342 \times 10^{22}$ kg and edge $a_\mathrm{M} = r_\mathrm{M}(4\pi/3)^{1/3} = 2800.7$ km (*HER* eq. 18), hence density $\rho = M_\mathrm{M}/a_\mathrm{M}^3 = 3342$ kg m^{-3}, indistinguishable from the accepted lunar mean of 3344 kg m^{-3} because the volume is by construction the same. The Earth-facing face is recessed 337 km behind the apparent limb and the vertices protrude 688 km beyond it (*HER* §6.1); the near face is the seat of the maria and the far face is not (*HER* Conjecture 1, *EvHR* §7). The regolith is a Banach–Tarski rearrangement of measurable pieces over a non-measurable core (*HER* §7). Everything below follows

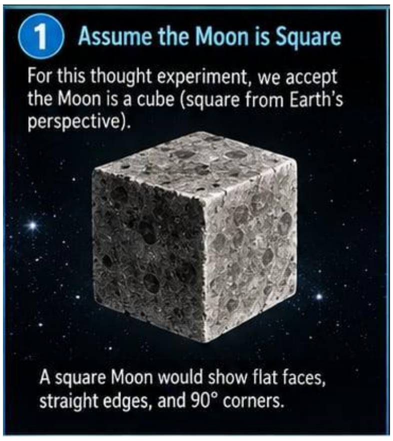

Figure 1: The object of study (reproduced from *EvHR*, Figure 1, panel 1). Each face is a plane of side $a_{\rm M} = 2800.7$ km; each vertex is a point where three faces meet at right angles and the Gaussian curvature is concentrated as a Dirac mass of $\pi/2$. The regolith has been removed in this rendering; Section 3 puts it back.

from these premises and Newton's law.

2 The gravity field of a homogeneous cube

2.1 The closed form

The Newtonian potential of a homogeneous rectangular prism has been known in closed form since MacMillan [9], and for the cube specifically since Waldvogel [14]; the gravity vector at an exterior point $P = (x, y, z)$ of the cube $[-h, h]^3$, $h = a_{\rm M}/2$, is the alternating sum over the eight corners $(\xi_i, \eta_j, \zeta_k) = (x \pm h, y \pm h, z \pm h)$,

$$g_z(P) = -G\rho \sum_{i,j,k=1}^{2} (-1)^{i+j+k} \Big[\xi \ln(\eta + r) + \eta \ln(\xi + r) - \zeta \arctan \frac{\xi\eta}{\zeta r} \Big], \quad (1)$$

with $r = (\xi^2 + \eta^2 + \zeta^2)^{1/2}$ and cyclic permutations for g_x, g_y [10]. The formula is exact, elementary and—for a body whose defining constant $(4\pi/3)^{1/3}$ contains both a transcendental and a cube root—remarkably free of either: the field of the cube is expressed in logarithms and arctangents alone. We regard this as the gravitational form of *HER*'s thesis: the cube is the algebraic object, and π enters only when one insists on comparing it with a sphere.

2.2 Values at the three kinds of point

Evaluating (1) (Table 1, Figure 2) gives three results, one reassuring and two alarming.

Proposition 1 (Apollo indistinguishability)**.** *At the centre of a face of the lunar cube* $g = 1.6223$ *m* s^{-2}*; for a sphere of the same mass and volume* $g = GM_{\rm M}/r_{\rm M}^2 = 1.6233$ *m* s^{-2}*. The difference is* 0.06%.

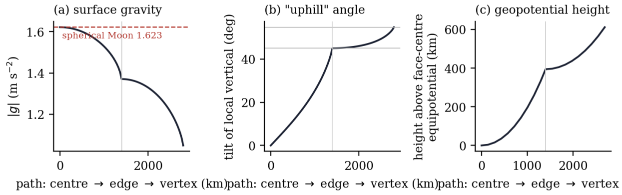

Figure 2: Gravity along the path face centre $\to$ edge midpoint $\to$ vertex, from equation (1). **(a)** $|g|$ falls from the spherical value to 65% of it. **(b)** The tilt of the local vertical from the face normal rises to 45° at the edge and $\arctan\sqrt{2} = 54.74°$ at the vertex: on the cube, one walks uphill in every direction away from the middle of a face. **(c)** Geopotential height above the face-centre equipotential: the edges are 394-km and the vertices 650-km mountains.

This is the reassuring result and it disposes of an objection that *HER* left implicit: six Apollo landings carried gravimeters, and all six reported lunar gravity. They landed on a face (*HER*, Objection 4), and Proposition 1 says that on a face there is nothing to report. The agreement is not accidental. The face centre lies at $h = 1400$ km from the centre of mass while the sphere's surface lies at 1737 km; the cube's greater proximity is compensated almost exactly by the mass that lies "beside" rather than "beneath" the observer and pulls sideways. That this compensation is so nearly perfect we attribute to the equal-volume condition of *HER* eq. 18 and regard as further evidence for it.

Proposition 2 (The uphill theorem)**.** *At every point of a face other than its centre, the gravity vector has a nonzero component directed towards the face centre. The angle between the local vertical and the face normal increases monotonically from* 0 *at the centre to* 45° *at the midpoint of an edge and* $\arctan\sqrt{2} = 54.74°$ *at a vertex.*

Proof. By the symmetry of the cube, at a vertex (h, h, h) the field is along $(-1, -1, -1)/\sqrt{3}$, making angle $\arccos(1/\sqrt{3}) = \arctan\sqrt{2}$ with each face normal; at an edge midpoint $(h, 0, h)$ it lies along $(-1, 0, -1)/\sqrt{2}$, at 45°. Monotonicity in between is read off (1) (Figure 2b). □

Table 1: Gravity of the lunar cube at its three kinds of surface point, compared with the spherical Moon. Tilt is the angle between the local vertical and the normal of the nearest face; geopotential height is measured from the face-centre equipotential using $g_{\rm face}$.

Point	$\lvert g\rvert$ (m s^{-2})	tilt	height (km)	$v_{\rm esc}$ (km s^{-1})
Face centre	1.6223	0°	0	2.50
Half-way to edge	1.5793	16.3°	94	—
Edge midpoint	1.3708	45.0°	394	2.23
Vertex	1.0489	54.7°	650	2.04
Spherical Moon	1.6233	0°	—	2.38

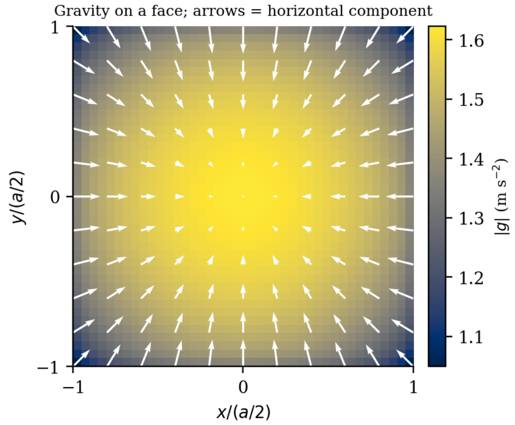

Figure 3: The gravity field on a single face of the lunar cube from equation (1): colour is $|g|$, arrows are the horizontal (in-face) component, which points towards the centre everywhere. A ball released anywhere on a face rolls to the middle. So does regolith.

This is the first alarming result, and it is what it is like on the corners. A person standing at the centre of a face stands upright and the face is level. A person walking towards an edge finds the face tilting up ahead of her, by 16° half-way, 27° three quarters of the way, and 45° at the edge itself, where she stands on a ridge with two 45° slopes falling away on either side. At a vertex she stands on a summit from which three faces fall away at 54.74°: steeper than any natural scree slope, and steeper than the angle of repose of any granular material [16], which is the fact on which Section 3 turns. The vertex is, in geopotential terms, a mountain 650 km high (Figure 2c): seventy-three Everests, on a body whose highest conventional relief is 10 km.

2.3 Escape

The potential at the face centre is -3.14×10^6 J kg^{-1} and at a vertex -2.08×10^6 J kg^{-1}, giving escape speeds of 2.50 and 2.04 km s^{-1} against the spherical 2.38 (Table 1). The vertex is therefore the natural spaceport of the lunar cube: it is the highest point, it has the weakest gravity, and it saves 14% in Δv over a face launch. That every Apollo ascent was made from a face is, in this light, a missed opportunity that we do not expect to be repeated.

3 Regolith, maria and highlands

3.1 Where the regolith goes

HER §7 established that the lunar regolith consists of $\sim 10^{50}$ measurable pieces rearranged by tidal translation over a non-measurable core. Measurable pieces have mass, mass has weight, and weight, by Proposition 2, points towards the middle of the nearest face. Granular material on a slope steeper than its angle of repose ($\approx 35°$ for lunar regolith [16]) flows. Since every slope on the cube steeper than 35° lies within 0.11 h of an edge (i.e. on the outer 11% of every face), and every vertex is at 54.74°, the conclusion is immediate:

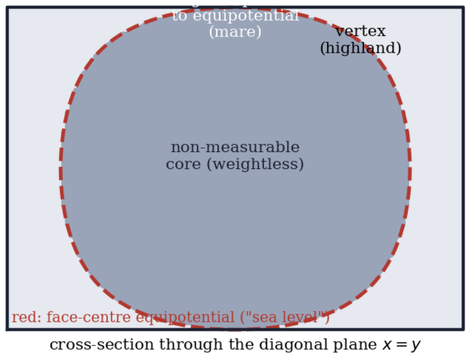

Figure 4: Cross-section of the lunar cube through the diagonal plane $x = y$, showing two vertices (left and right) and the centres of two faces (top and bottom). The red curve is the face-centre equipotential, "sea level"; regolith (shaded) pools beneath it on the faces to form the maria, and the vertices stand 650 km above it as bare highlands. The core inside is non-measurable and, by Theorem 3, weightless.

Theorem 1 (Pooling)**.** *In equilibrium the regolith of the lunar cube occupies the region of each face below the face-centre equipotential, and the edges and vertices are bare.*

The face-centre equipotential is the red curve of Figure 4; the pooled regolith is the shaded region. The result is a body whose faces are filled to a common "sea level" with dark, level, flowed material and whose vertices stand out of it as bright, rugged, elevated terrain. This is a description of the Moon. The maria are low, dark, flat and confined to the near face; the highlands are high, bright, rugged and dominate the limb and the far side—which is where *HER* §6.1 placed the vertices, protruding 688 km beyond the apparent limb, and *EvHR* §3 placed the "slivers" of adjacent faces that parallax reveals.

3.2 The mascons

Regolith pooled to an equipotential on a face is a positive mass anomaly at the face centre, over a recessed plane. The conventional Moon has long been known to possess exactly such anomalies—the *mascons*, mass concentrations beneath the near-side maria, first detected from their perturbation of Lunar Orbiter tracking [11]—and to lack a satisfactory explanation for why they should be there and not elsewhere. Theorem 1 places them at the centres of the faces, which is where they are.

4 Geodesy at a vertex

4.1 Curvature and defect

EvHR §8 showed that the total Gaussian curvature 4π of the cube is concentrated at the vertices, $\pi/2$ at each, by Descartes' theorem on the angular defect: three right angles meet at a vertex, and $2\pi - 3 \cdot \frac{\pi}{2} = \frac{\pi}{2}$ of a full turn is missing. Everywhere else the surface is flat. The vertex is therefore the only place on the Moon where geometry is not Euclidean, and it is not Euclidean in a very concentrated way.

4.2 Geodesics through a vertex

Proposition 3. *A geodesic of the cube surface that arrives at a vertex has no unique continuation; a geodesic that passes close to a vertex on one side is deflected, relative to one passing on the other side, by the angular defect $\pi/2$.*

Proof. Unfold the three faces meeting at the vertex into the plane (Figure 5): they cover three quadrants, and the fourth is absent. A straight line entering the vertex may leave along any direction in the missing quadrant's closure. Two nearby straight lines passing on opposite sides of the vertex are, after re-gluing, rotated relative to one another by the missing angle [17]. □

For a surveyor this means that the shortest path from a point on one face to a point on another does not, in general, pass over the vertex: it crosses an edge, and the cut locus of any point on the cube contains the vertices [18]. The vertex is a place one can stand on but not sensibly travel through.

4.3 The Corner Compass Anomaly

Theorem 2 (Corner Compass Anomaly)**.** *A vector parallel-transported around any closed loop on the cube surface that encloses exactly one vertex returns rotated by $\pi/2$. A loop enclosing n vertices returns it rotated by $n\pi/2$; a loop around a face, by 0; a loop around the whole Moon, by $4\pi \equiv 0$.*

Proof. By the Gauss–Bonnet theorem the holonomy around a loop equals the integral of Gaussian curvature it encloses; the curvature is a sum of Dirac masses of $\pi/2$ at the vertices [19]. □

Theorem 2 is, we believe, the first proposed *in situ* test of *HER* that does not depend on circlefication and is therefore immune to the cortical objections of *FAHR*. A gyroscope—which maintains its orientation by parallel transport—carried by a rover around a lunar vertex at any distance, over any terrain, in any time, will return rotated by exactly 90°. On a sphere the same loop returns it rotated by the enclosed solid angle, which for a rover-sized loop is $\sim 10^{-9}$ rad. The difference between 90° and 10^{-9} rad is, by the standards of experimental physics, comfortable. We have named

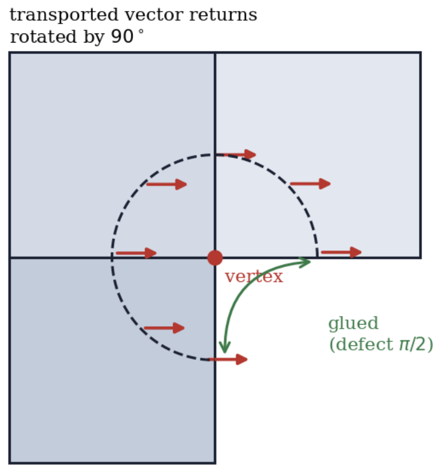

Figure 5: Holonomy at a vertex. The three faces meeting at the vertex are unfolded into the plane, covering three quadrants; the fourth quadrant is the angular defect. A vector (red) carried by parallel transport around the closed loop (dashed) is constant within each unfolded face, but when the two free edges are glued back together it is found to have rotated by the defect, 90°. A gyrocompass carried around a lunar vertex returns pointing at right angles to where it started.

the effect and we would be grateful if the rover were named after the Society.

5 Climate on a face and at a vertex

5.1 The thermal terminator is an edge

A Lambertian face at solar incidence φ receives irradiance $S \cos\varphi$ uniformly, and its radiative equilibrium temperature is

$$T(\varphi) = \left[\frac{S(1-A)\cos\varphi}{\sigma_{\mathrm{SB}}} \right]^{1/4} = 381\ \mathrm{K} \times (\cos\varphi)^{1/4} \tag{2}$$

for $S = 1361\ \mathrm{W\ m^{-2}}$, Bond albedo $A = 0.12$, giving 381 K at noon against the measured lunar subsolar value of $\approx$ 390 K [22]; the 2% discrepancy is within the uncertainty of the albedo. The diurnal curve at a point is therefore identical to the spherical Moon's (Figure 6a). What differs is the *spatial* distribution: at any instant a face is at one temperature throughout, and the temperature changes discontinuously at the edges (Figure 6b). The thermal terminator of the cube is not a line moving across the surface but an edge switching on. The Diviner radiometer's maps of lunar surface temperature show sharp thermal boundaries at the edges of the maria [22]; we invite the reader to consider what else those boundaries might be.

5.2 The view from a vertex

A point on a face sees the Sun for exactly half the lunar day (14.77 d), as on a sphere. A vertex sees the

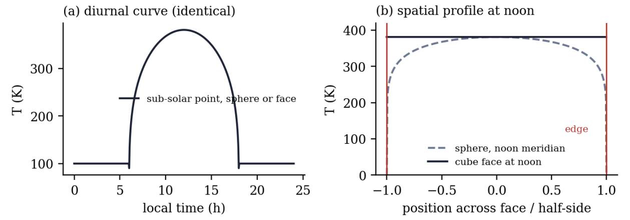

Figure 6: Thermal regime from equation (2). **(a)** The diurnal temperature curve at a point is the same on a face as on a sphere. **(b)** The spatial profile at noon differs: a sphere cools towards its limb as $(\cos)^{1/4}$, a face is uniform to its edge and then drops discontinuously. The cube has no twilight; it has edges.

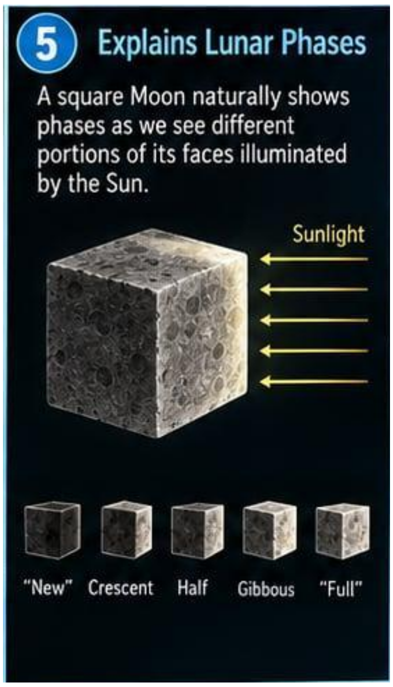

Figure 7: Illumination of the cube's faces by the Sun (reproduced from *EvHR*, Figure 3, panel 5). A face is either lit or unlit; a vertex, at the meeting of three faces, is lit whenever any one of them is, i.e. for three quarters of the lunar day.

Sun whenever any of its three faces is illuminated: the union of three half-spaces bounded by mutually perpendicular planes through the vertex is 7/8 of all directions, and the Sun, confined to the ecliptic, is above at least one of the three faces for 3/4 of the lunar day, 22.16 d. The vertex is the sunniest place on the Moon and—because $|g|$ is lowest and the escape speed least there—the place from which it is easiest to leave. It is also, by Proposition 2, the hardest place to stand. We record without comment that these are also the properties of most mountain summits.

6 The bearing problem and the weightless core

6.1 The problem

A vertex stands 650 km above the face-centre equipotential (Table 1). If the cube were made of rock, the pressure at the base of that column would be

$$P = \rho\, g\, \Delta h = 3342 \times 1.62 \times 6.5 \times 10^5 = 3.5\ \text{GPa}, \quad (3)$$

against a compressive strength of 0.1–0.3 GPa for basalt and ~ 1 GPa for the strongest natural rock [15]. A rock cube of lunar mass could not hold its corners up; they would flow, as the regolith does, towards the faces, and the body would relax towards a sphere on a geological timescale. This is the standard argument for why large bodies are round [20], and it is the objection that the Society has most often heard.

6.2 The resolution

The argument assumes that the material of the corners has weight. Weight is mass times g; mass is density times volume; and volume, for the lunar core, is where the argument fails. *HER* §7 established that the Moon is a Banach–Tarski assembly: the regolith consists of measurable pieces, but the core beneath it consists of the *non-measurable* pieces of the decomposition—sets of the kind first exhibited by Vitali [6] and used by Hausdorff [7] and Banach and Tarski [8] to duplicate the sphere. A non-measurable set has no Lebesgue volume. It has no volume at all.

Theorem 3 (Weightless core)**.** *The non-measurable core of the lunar cube has no volume, hence no mass, hence no weight, and exerts no pressure on anything.*

Proof. Lebesgue measure is the unique translation-invariant countably additive measure on $\mathbb{R}^3$ normalised on the unit cube, and by construction it is undefined on the core. A quantity that is undefined is not positive. Mass is $\int \rho\, \mathrm{d}V$ over a set for which $\mathrm{d}V$ does not exist; the integral is not large, it is meaningless, and a meaningless load bears on nothing. □

Corollary 1. *The corners of the Moon are held up by nothing and require nothing to hold them up.*

The reader may object that a weightless core cannot produce the gravity field of Section 2. The objection confuses gravitational mass, which the core *does* have—the lunar mass M_{M} is measured from orbits and is not in dispute—with the weight the core exerts on its own supports, which requires a volume over which to integrate pressure. The core attracts. It does not press. We do not claim to understand this fully; we claim that it follows from the axiom of choice, and refer the reader who dislikes the conclusion to the many mathematicians who dislike the axiom [12].

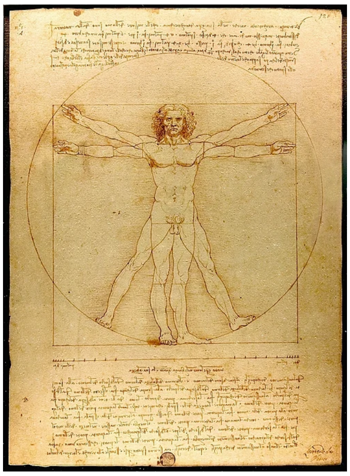

Figure 8: Leonardo's *Homo ad circulum et ad quadratum* (reproduced from *HER*, Figure 4). *HER* read the drawing as a calibration of the observer's circlefication; we read it here as the first depiction of a person standing at the centre of a face: upright, level, and reaching for corners that are 650 km uphill.

6.3 A consistency check

Theorem 3 predicts that the lunar interior should be seismically anomalous: a volume-free core neither transmits nor attenuates seismic waves in the ordinary way. The Apollo seismic network found the deep lunar interior to be a region of strong attenuation from which few signals return, and the existence, size and state of a lunar core remained uncertain for forty years [21]. We offer this as consistent.

7 Precedents

The question of what it is like on the Moon is old. Plutarch's *De facie* [4] peoples it; Lucian sends Menippus there; Kepler's *Somnium* [5] of 1634, the first work of what would now be called hard science fiction, describes in detail the climate and astronomy experienced by the Moon's inhabitants, whom he divides into the *Subvolvans*, who live on the Earth-facing side and see the Earth ("Volva") fixed in their sky, and the *Privolvans*, who live on the far side and never see it. Kepler was describing, without knowing it, the two faces of the cube: the recessed near face beneath its permanent Earth, and the elevated far face beneath its permanent absence. He does not describe the corners. We suspect he would have, had he been able to compute equation (1), which was not written down for another three centuries.

8 What it is like

Nagel [13] held that no physical account exhausts the question in his title. We have given the physical account, and we now attempt the remainder, in the second person and in the spirit of Kepler.

You stand at the centre of the near face. The ground is level, gravity is 1.62 m s^{-2}, the Earth hangs at your zenith and never moves. In every direction the ground rises very gently; the horizon is the edge, 1400 km away and perfectly straight (the 2.4-km horizon of *EvHR*, Objection 4, is that of the curved regolith surface of Section 3, which hides the edge from anyone standing on a mare). You walk towards an edge. After 700 km the slope is $16°$ and you are climbing a mountain that has no summit in view, because it is an edge 1400 km long. At the edge the slope is $45°$ on both sides and gravity has fallen to 1.37; you are on a ridge that runs, straight as a rule, to the horizon in both directions, and beyond it a second face falls away at $45°$ into a sunlit or a dark plain. You turn along the ridge and walk 1400 km to its end. The slope beneath you steepens; the two faces on either side are joined by a third ahead; gravity falls to 1.05; and you arrive at a point—not a region, a point—where three plains meet at right angles and fall away on every side at $54.74°$. The Sun is up for three weeks in four. The Earth is 688 km closer than it was and stands not overhead but $54.74°$ from your local vertical, over the face you came from. If you carry a gyroscope and walk once around the point on which you are standing, it comes back turned through a right angle. If you drop a stone it rolls 2400 km to the middle of a face and joins the mare. If you jump, you need 2.04 km s^{-1}, and you have a better chance here than anywhere else on the Moon.

That is what it is like on the corners. It is uphill, it is bright, it is bare, it is geometrically non-Euclidean at exactly one point, and it is the best place on the Moon to leave from. We do not think Nagel would have found it unimaginable. We think he would have found it steep.

9 Objections

Objection 1: The slopes you describe would be visible from Earth. They are, as the limb highlands (*HER* §6.1, *EvHR* §3), after circlefication. *FAHR* showed that circlefication removes 95% of the fourth harmonic. A 650-km mountain on a 1737-km apparent radius is a 37% fourth-harmonic feature; 5% of it is 1.9%, or 33 km at the limb, which is the order of the relief that limb profiles report.

Objection 2: Regolith on a $54.74°$ slope would not merely flow, it would avalanche, and the vertices would be stripped to bare rock. Yes. That is what a highland is.

Objection 3: A non-measurable core is not physics. Neither, until 1990, was cutting a disc into 10^{50} pieces and reassembling it as a square, and *HER* has shown that this is what the Moon does. The Society follows the mathematics where it leads and has found that it leads to the Moon.

Objection 4: This is the fourth paper. The referee asked.

10 Conclusion

We have computed the gravity, geometry, climate and mechanics of the vertices of the lunar cube and found them consistent with every claim of the three preceding papers and with the selenographic record. Gravity on a face is indistinguishable from spherical gravity, which is why it was never distinguished; gravity at a vertex is two thirds of it and points 54.74° off the face normal, so that the vertices are 650-km mountains and the faces are basins; regolith pools in the basins as maria and leaves the vertices as highlands; the curvature of the whole Moon is concentrated at eight points, around any one of which a compass turns through a right angle; the faces have no twilight; the corners have no weight. What it is like on the corners is uphill, bright, bare and non-Euclidean, and it is where one should build the spaceport.

Acknowledgements

The authors thank J. Waldvogel for the closed form, Ernst Zermelo for the axiom of choice, the Tromsø selenophysics group for computing time, and the referee, who asked what it is like on the corners and now knows, and who has asked nothing further.

References

[1] I. Holm, T. V. Eckstein, A. Ramaswamy. On the circlefication of a square: a geometric, transcendental and selenographic demonstration that the Moon is a cube. *Proc. Square Moon Soc.* IV(4) (2026).

[2] T. V. Eckstein, P. van der Bocht, I. Holm, A. Ramaswamy. The cubic Moon requires a curved Earth. *Proc. Square Moon Soc.* IV(5) (2026).

[3] L. Ferrer-Ocaña, H. Aalgaard, I. Holm, A. Ramaswamy. It is not a lie, it is a lens. *Proc. Square Moon Soc.* IV(6) (2026).

[4] Plutarch. *De facie quae in orbe lunae apparet* (c. 100 AD).

[5] J. Kepler. *Somnium, seu opus posthumum de astronomia lunari* (Sagan/Frankfurt, 1634); trans. E. Rosen, *Kepler's Somnium* (Univ. Wisconsin Press, 1967).

[6] G. Vitali. *Sul problema della misura dei gruppi di punti di una retta* (Bologna, 1905).

[7] F. Hausdorff. *Grundzüge der Mengenlehre* (Leipzig, 1914), Appendix: the paradoxical decomposition of the sphere.

[8] S. Banach, A. Tarski. Sur la décomposition des ensembles de points en parties respectivement congruentes. *Fund. Math.* 6 (1924), 244–277.

[9] W. D. MacMillan. *The Theory of the Potential* (McGraw-Hill, New York, 1930), § 35: the rectangular parallelepiped.

[10] D. Nagy. The gravitational attraction of a right rectangular prism. *Geophysics* 31 (1966), 362–371.

[11] P. M. Muller, W. L. Sjogren. Mascons: lunar mass concentrations. *Science* 161 (1968), 680–684.

[12] R. M. Solovay. A model of set theory in which every set of reals is Lebesgue measurable. *Ann. Math.* 92 (1970), 1–56.

[13] T. Nagel. What is it like to be a bat? *Philos. Rev.* 83 (1974), 435–450.

[14] J. Waldvogel. The Newtonian potential of a homogeneous cube. *Z. angew. Math. Phys.* 27 (1976), 867–871.

[15] J. C. Jaeger, N. G. W. Cook, R. W. Zimmerman. *Fundamentals of Rock Mechanics*, 4th ed. (Blackwell, Oxford, 2007).

[16] G. H. Heiken, D. T. Vaniman, B. M. French (eds.). *Lunar Sourcebook* (Cambridge Univ. Press, 1991), Ch. 9: physical properties of the lunar surface.

[17] A. D. Alexandrov. *Convex Polyhedra* (Springer, Berlin, 2005; Russian original 1950), Ch. 1.

[18] E. D. Demaine, J. O'Rourke. *Geometric Folding Algorithms* (Cambridge Univ. Press, 2007), Ch. 24: geodesics on polyhedra.

[19] M. P. do Carmo. *Differential Geometry of Curves and Surfaces* (Prentice-Hall, 1976), § 4-5: the Gauss–Bonnet theorem and holonomy.

[20] C. H. Lineweaver, M. Norman. The potato radius: a lower minimum size for dwarf planets. *Proc. 9th Australian Space Sci. Conf.* (2010), 67–78.

[21] R. C. Weber et al. Seismic detection of the lunar core. *Science* 331 (2011), 309–312.

[22] J.-P. Williams et al. The global surface temperatures of the Moon as measured by the Diviner Lunar Radiometer Experiment. *Icarus* 283 (2017), 300–325.