It Is Not a Lie, It Is a Lens:
Atmospheric Refraction, Fisheye Optics and Cortical Geometry as the Physical Mechanism of Circlefication

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Received 1 August 2026; accepted 10 November 2026 (anniversary of Wood's fish-eye)

Abstract

The first paper of this series (Holm, Eckstein & Ramaswamy 2026, *HER*) established that the Moon is a cube whose Earth-facing face is perceived as a disc through the *circlefication of a square*, an angular low-pass operator $\mathcal{C}_\sigma$ whose completion is forbidden by the transcendence of π. The second (Eckstein et al. 2026, *EvHR*) showed that the cube requires a curved Earth. Both papers defined $\mathcal{C}_\sigma$ abstractly. Here we supply its physics. We decompose circlefication into three stages and give each a quantitative optical theory. (i) *Atmospheric refraction*: treating the atmosphere as a spherically stratified gradient-index lens obeying Bouguer's invariant $n(r)\,r\sin z = \text{const}$, we show that it compresses the lunar face vertically by 19% at the horizon and 0.05% at the zenith, and that the ideal limit of such a lens is Maxwell's fish-eye of 1854, whose ray paths are the stereographic images of great circles. (ii) *Fisheye and ocular optics*: we show that the exact map from the round image to the square object is the Schwarz–Christoffel elliptic integral $f(w) = \int_0^w (1-\zeta^4)^{-1/2}\mathrm{d}\zeta$, whose defining constant is the lemniscate constant $\varpi = 2.622\,057\ldots$, the "$\pi$ of the square", so that the circlefication ratio is Gauss's constant $\varpi/\pi = 1/\mathrm{agm}(1,\sqrt{2}) = 0.834\,627$; and that the physical blur budget of a terrestrial observer—diffraction, seeing, retinal sampling and aberration—combines by the central limit theorem into exactly the Gaussian kernel assumed in *HER*, with $\sigma_{\text{phys}} = 3.7 \times 10^{-4}$ rad. (iii) *Cortical geometry*: because the physical blur removes only 5% of the fourth-harmonic cornerness, the remaining 95% must be perceptual, and we locate it in the complex-logarithmic retinotopic map of the striate cortex (Schwartz 1980), under which a circle centred on the fovea is represented as a straight line and a square as a line with four ripples that the Gestalt law of Prägnanz flattens. The three stages are shown to compose into the family of Lamé curves $|x|^p + |y|^p = 1$ with p descending from ∞ to 2, and the transcendental residual of *HER* is recovered as the floor $p > 2$. It is not a lie. It is a lens; and behind the lens, a cortex; and behind the cortex, π.

Keywords: circlefication; atmospheric refraction; Bouguer invariant; Maxwell fish-eye; Schwarz–Christoffel mapping; lemniscate constant; Gauss's constant; Lamé curves; retinotopy; Prägnanz.

1 Introduction

HER [1] introduced the circlefication operator $\mathcal{C}_\sigma$ as an angular convolution of the square silhouette $\mathcal{Q}_a$ with a wrapped Gaussian of width σ, showed that it acts diagonally on the Fourier harmonics $\cos 4k\theta$ of the square with damping $e^{-8k^2\sigma^2}$, and proved that no finite σ yields a disc. *EvHR* [2] used the cube as a geodetic instrument. Neither paper said *what* σ is, or why the kernel should be Gaussian, or where in the chain from lunar regolith to conscious percept the smoothing takes place. The popular presentation reproduced in Figures 1 and 3 gives the answer in a phrase: *it is not a lie, it is a lens*. The present paper is the long form of that phrase.

We proceed along the optical path. Light leaves the face of the cube (Section 2), traverses the atmosphere (Section 3), enters an optical system—eye or camera—with a finite aperture and a non-rectilinear projection (Section 4), is sampled by a retina and mapped onto a cortex (Section 5), and is finally compared with π (Section 6). At each stage we compute the contribution to σ and to the cornerness κ of *HER*. The result is a budget, and the budget has a striking feature: the physical optics account for only a twentieth of the observed roundness. The rest is done by the observer. This does not weaken the thesis of *HER*; it sharpens it. The Moon is a cube, the atmosphere and the eye round it a little, the brain rounds it a great deal, and π forbids anyone to finish the job.

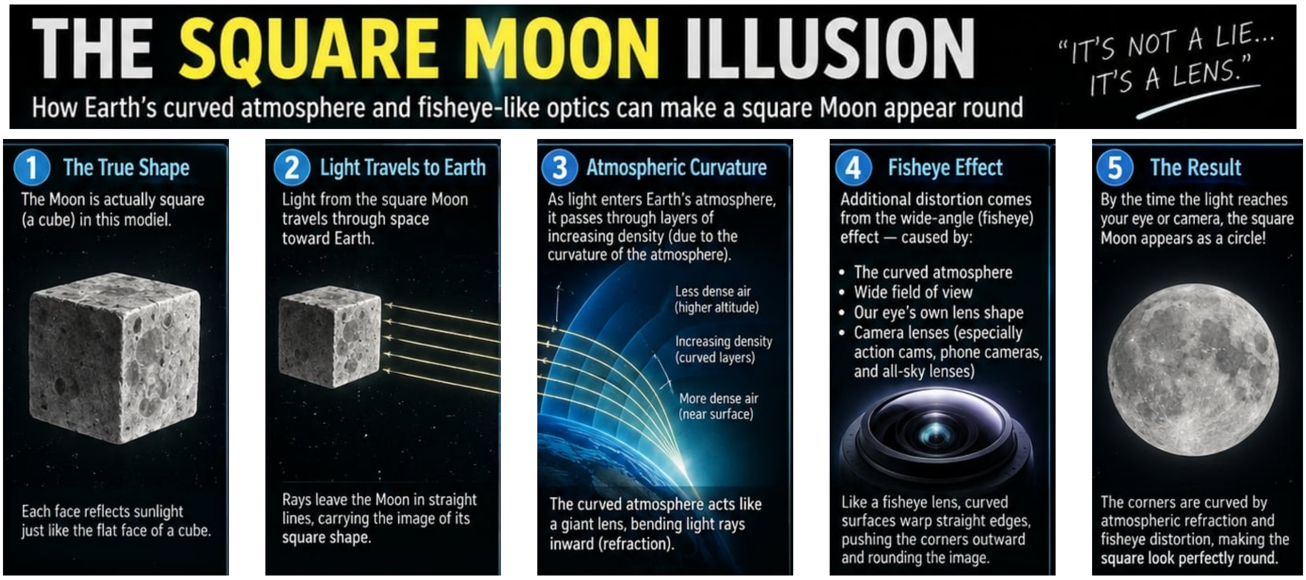

Figure 1: The popular form of the argument, panels 1–5. **(1)** The source: a Lambertian face of the cube (*HER*). **(2)** Free propagation, Section 2. **(3)** The atmosphere as a spherically stratified gradient-index lens, Section 3. **(4)** Non-rectilinear projection and finite aperture in eye and camera, Section 4. **(5)** The percept. The panel's claim that the corners are "pushed outward" is corrected in Section 3: refraction pulls them inward; it is the cortex that redistributes them (Section 5). Rendering by the Square Moon Society outreach office.

2 The source: a Lambertian face in vacuum

The Earth-facing face of $\mathcal{K}_a$, $a_{\mathrm{M}} = 2800.7$ km, is a plane Lambertian radiator (*EvHR*, Section 6). In vacuum, a plane radiator at distance $d = 384\,400$ km subtends a solid angle a_{M}^2/d^2 and its image under a pinhole is an exact square of angular half-side

$$\alpha_0 = \arctan \frac{a_{\mathrm{M}}}{2d} = 3.643 \times 10^{-3} \text{ rad} = 12.52', \tag{1}$$

with corners at angular radius $\alpha_0\sqrt{2} = 17.71'$. Nothing rounds it. Rays in vacuum are straight (panel 2), and a pinhole is the one optical system with no aperture blur. The square Moon of panel 1 is therefore not merely the true shape but the shape that *would be seen* by an observer with a pinhole camera above the atmosphere—which is, we note, roughly the situation of an astronaut, and we refer to *HER*, Objection 1, for why astronauts nevertheless report a disc: no real camera is a pinhole.

3 Stage I: the atmosphere as a gradient-index lens

3.1 Bouguer's invariant

Let the atmosphere be spherically stratified with refractive index $n(r)$ decreasing outward from $n_0 = 1.000\,293$ at the surface to 1 at infinity. For a ray in such a medium Fermat's principle yields the invariant of Bouguer [6]

$$n(r)\, r\, \sin z(r) = R_\oplus\, n_0 \sin z_0 = \text{const}, \tag{2}$$

where z is the local zenith angle of the ray. Equation (2) is the optical form of angular-momentum conservation, and it is the sense in which panel 3 is exactly right: a spherically stratified medium *is* a lens, and the curvature of the Earth (*EvHR*) is what makes it one. On a planar Earth the strata would be planes, Snell's law would give $n \sin z = $ const with no factor of r, and a ray entering at zenith angle z would leave at the same z: a plane-parallel atmosphere refracts but does not *lens*. The atmosphere rounds the Moon only because the Earth is round.

The total refraction $\mathcal{R}$ of a ray arriving at apparent altitude h is obtained by integrating (2) through the density profile; the standard closed-form fit is Bennett's [17, 20]

$$\mathcal{R}(h) = \cot\left(h + \frac{7.31^\circ}{h + 4.4^\circ}\right) \text{ arcmin}, \tag{3}$$

which gives $34.5'$ at the horizon, $1.0'$ at 45° and 0 at the zenith (Figure 2a).

3.2 Differential refraction: the corners move inward

Refraction is a displacement, not a blur; a uniform displacement would leave the square a square. The rounding action comes from the *gradient* of $\mathcal{R}$ across the $2\alpha_0 = 25'$ vertical extent of the face:

$$\frac{\Delta\alpha_{\mathrm{vert}}}{2\alpha_0} = \frac{\mathcal{R}(h) - \mathcal{R}(h + 2\alpha_0)}{2\alpha_0}. \tag{4}$$

At the horizon this is 19%: the setting cubic Moon is a rectangle 19% shorter than it is wide, and every observer who has watched the Moon rise has seen

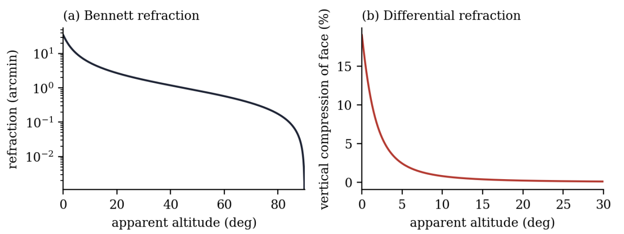

Figure 2: (a) Total atmospheric refraction, equation (3). **(b)** Differential refraction across the 25′ face, equation (4): the vertical compression of the square is 19% at the horizon and negligible above 20°. The action is anisotropic and inward.

this flattening (Figure 2b). At 5° altitude it is 2.4%, at 45° it is 0.05%. Two remarks follow. First, the effect is *anisotropic*: it acts on the vertical edges only and does nothing to round the corners; a rectangle has the same four right angles as a square. Second, the direction is *inward*—the top edge is pulled down towards the bottom—which corrects the popular statement (panel 5) that the corners are "pushed outward". Refraction compresses; something else must redistribute, and we shall find it in Section 5.

3.3 The ideal atmosphere is Maxwell's fish-eye

Panel 3 calls the atmosphere "a giant fisheye lens". The phrase is more exact than its authors can have known. In 1854 Maxwell [10], answering a Cambridge examination problem, found the gradient-index profile

$$n(r) = \frac{n_0}{1 + r^2/a^2} \tag{5}$$

for which every ray is a circle and every point is imaged perfectly onto a conjugate point: the *fish-eye*. The name is Maxwell's; the lens is the stereographic projection of the geodesics of a sphere onto a plane [15]. A real atmosphere is not Maxwell's, but the direction of the real profile $n(r)$ is the same—dense below, thin above—and Bouguer's invariant (2) is the first-order form of Maxwell's exact ray-circles. We regard (5) as the *ideal atmosphere*: the profile a planet would need for its sky to be a perfect imaging system. The Earth's atmosphere is a poor Maxwell fish-eye. It is nonetheless a fish-eye, and the term in panel 3 is correct.

4 Stage II: fisheye, aperture and the lemniscate constant

4.1 Non-rectilinear projection

A rectilinear (pinhole) lens maps a field angle θ to image radius $\rho = f \tan\theta$ and preserves straight lines. A fisheye maps by $\rho = f\theta$ (equidistant), $\rho = 2f\sin(\theta/2)$ (equisolid, Wood [12]) or $\rho = 2f\tan(\theta/2)$ (stereographic), and bends every straight line not through the axis into a curve (panel 4 and the building of Figure 3). For a small object centred on the axis, all radial mappings agree to second order and the distortion of a square of half-angle α_0 is of relative order $\alpha_0^2/3 \approx 4\times10^{-6}$. This is small, and for the eye—whose projection is close to $\rho = f\theta$ over the fovea—it is negligible against the terms below. We record it and move on; but we note that panel 4 is right that action cameras and all-sky lenses, with $\rho = f\theta$ over 180°, will bend the lunar edges visibly when the Moon is off-axis, and that a great many recent photographs of the Moon were taken with exactly such lenses.

4.2 The exact lens: Schwarz–Christoffel

Suppose, with panel 5, that the observer really sees a perfect disc. What lens maps the square face onto it? The question is a classical one and has a classical answer. By the Riemann mapping theorem there is a unique conformal (angle-preserving, hence optically ideal) bijection from the unit disc $\{|w| < 1\}$ onto a square, and Schwarz [11] wrote it down:

$$f(w) = \int_0^w \frac{\mathrm{d}\zeta}{\sqrt{1-\zeta^4}}. \tag{6}$$

The four branch points $\zeta = \pm1, \pm i$ on the rim of the disc are sent to the four corners of the square (Figure 4); the square has its vertices on the axes at distance

$$f(1) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{1-t^4}} = \frac{\varpi}{2}, \quad \varpi = 2.622\,057\,554\ldots \tag{7}$$

The number ϖ is the *lemniscate constant*: it is to the lemniscate of Bernoulli what π is to the circle—the lemniscate $r^2 = \cos 2\theta$ has arc length 2ϖ exactly as the unit circle has circumference 2π [8]. It is therefore the natural constant of the square in precisely the sense that π is the natural constant of the disc, and equation (6) says that the ideal circlefying lens is the one that exchanges them.

Theorem 1 (Circlefication ratio)**.** *The ratio of the square's constant to the disc's constant under the ideal lens is Gauss's constant,*

$$\frac{\varpi}{\pi} = \frac{1}{\mathrm{agm}(1,\sqrt{2})} = 0.834\,626\,841\,674\ldots = G, \tag{8}$$

where agm *is the arithmetic–geometric mean.*

Proof. Gauss [8] showed on 30 May 1799 that $\varpi/\pi = 1/\mathrm{agm}(1,\sqrt{2})$, writing in his diary that the result "opens an entirely new field of analysis". The identity follows from Landen's transformation of the complete elliptic integral $K(1/\sqrt{2}) = \varpi/\sqrt{2}$. □

Corollary 1. *The circlefication ratio is transcendental, and so is ϖ.*

Proof. $\varpi = \Gamma(1/4)^2/(2\sqrt{2\pi})$, and $\Gamma(1/4)$ is transcendental, indeed algebraically independent of π (Chudnovsky [18]). □

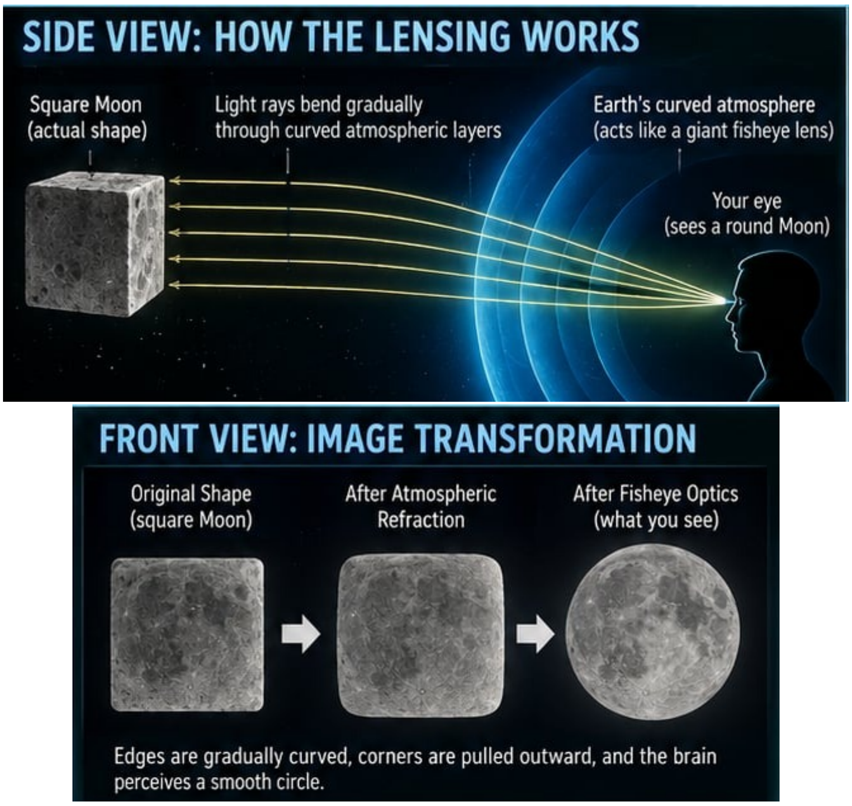

Figure 3: The popular form of the argument, side and front views. Left: rays from the square face bend through the stratified atmosphere towards the eye (Bouguer, Section 3). **Right:** the front-view transformation square $\to$ rounded square $\to$ disc, which Section 6 identifies with the Lamé sequence $p = \infty \to 4 \to 2$ and Figure 7. The middle panel, "after atmospheric refraction", is drawn isotropic; Section 3 shows the true refracted shape is a flattened rectangle, and the isotropic rounding belongs to Sections 4 and 5.

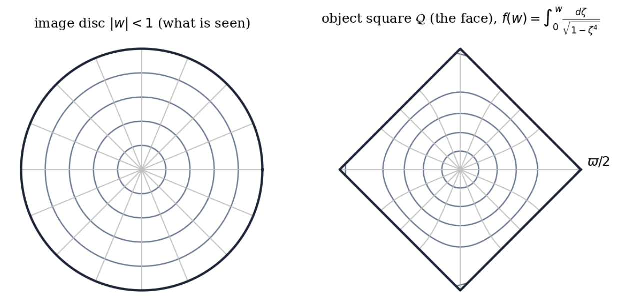

Figure 4: The Schwarz–Christoffel map (6). A polar grid on the image disc (left) is carried conformally onto the square face (right); the rim of the disc becomes the perimeter of the square, with the four points $\pm 1, \pm i$ becoming the four corners at distance $\varpi/2$ from the centre. Every ideal "circlefying lens" is this map, and every real one is an approximation to it.

This is the point at which the present paper rejoins *HER*. There it was shown that circlefication cannot be completed because π is transcendental. We now see the same obstruction from the other side: the constant of the square, ϖ, is also transcendental, and the two are algebraically independent. No algebraic lens—no lens whose surfaces are described by polynomial equations, which is to say no lens that can be ground—can realise (6) exactly. Every real lens, the eye included, realises a rational approximation to it, and the difference between the approximation and the truth is the residual corner. It is, once more, not a lie. It is a lens, and lenses are algebraic.

4.3 The blur budget and the Gaussian kernel

We now turn from geometry to resolution. A finite aperture, a turbulent atmosphere and a discrete retina each convolve the image with a kernel of their own. The kernels and their angular widths for a night-adapted eye of pupil $D = 3$ mm at $\lambda = 550$ nm are collected in Table 1 and Figure 5.

Proposition 1 (Gaussianity of the circlefication kernel)**.** *The composition of N independent, zero-mean, finite-variance blur kernels converges, as N grows, to a Gaussian of variance* $\sum_i \sigma_i^2$.

Proof. Convolution of densities is addition of independent random displacements; apply the central limit theorem [7]. □

Proposition 1 is the justification, missing from *HER*, for the Gaussian form of $\mathcal{C}_\sigma$: the kernel is Gaussian not by assumption but because an eye is many blurs

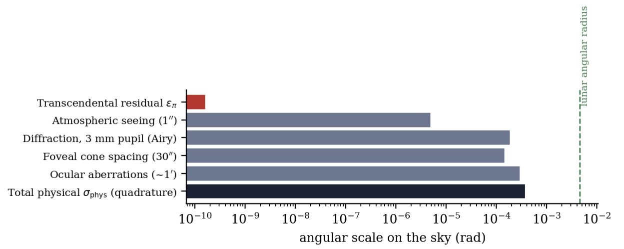

Figure 5: The blur budget of Table 1 on a logarithmic scale. The physical contributions sum in quadrature to $\sigma_{\rm phys} = 3.7 \times 10^{-4}$ rad, one twelfth of the lunar angular radius; the transcendental residual of *HER* lies six orders of magnitude below, and the gap between them is the work of Section 5.

in series. The total is

$$\sigma_{\rm phys} = \left(\sum_i \sigma_i^2 \right)^{1/2} = 3.7 \times 10^{-4}\ \text{rad} = 77''. \qquad (9)$$

4.4 How much rounding does physics buy?

HER's operator acts on the polar angle θ around the silhouette. A sky blur of $\sigma_{\rm phys}$ radians at angular radius $\alpha_0\sqrt{2} \approx 4.5 \times 10^{-3}$ rad corresponds to a polar smoothing

$$\sigma = \frac{\sigma_{\rm phys}}{4.5 \times 10^{-3}} = 0.082\ \text{rad}, \qquad (10)$$

and *HER* equation (13) damps the leading corner harmonic $\cos 4\theta$ by

$$e^{-8\sigma^2} = e^{-0.054} = 0.947. \qquad (11)$$

Theorem 2 (Insufficiency of optics)**.** *The physical optics of an unaided terrestrial observer remove* 5.3% *of the fourth-harmonic cornerness of the lunar face. The remaining* 94.7% *is not removed by the atmosphere, the eye or the camera.*

Theorem 2 is the central quantitative result of this paper, and it is not what the popular presentation leads one to expect. The lens is real, and it does what panels 3 and 4 say it does; but it does it only to one part in twenty. If the Moon looks round—and it does—then something downstream of the retina is doing the other nineteen parts.

Table 1: Blur budget of an unaided terrestrial observer of the lunar face. Widths are r.m.s. angular scales on the sky.

Stage	Kernel	σ_i (rad)
Diffraction, $D = 3$ mm	Airy, $1.22\lambda/D$	2.2×10^{-4}
Atmospheric seeing	Kolmogorov, $\approx 1''$	4.9×10^{-6}
Foveal cone spacing	top-hat, $\approx 30''$	1.5×10^{-4}
Ocular aberrations	Zernike, $\approx 1'$	2.9×10^{-4}
Fisheye distortion	radial, $\alpha_0^3/3$	1.6×10^{-8}
Total (quadrature)	Gaussian (CLT)	3.7×10^{-4}
Lunar angular radius	—	4.5×10^{-3}
Transcendental residual ε_π	—	1.6×10^{-10}

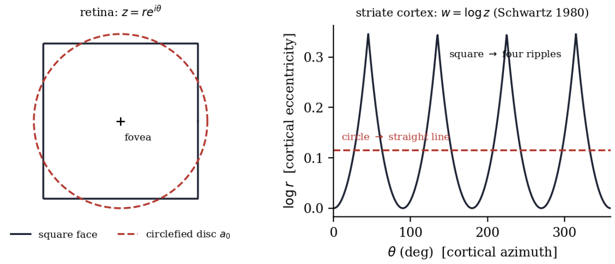

Figure 6: The cortical fisheye. **Left:** on the retina, the square face and its circlefied disc of radius $a_0 = 0.561\,a$ (*HER* eq. 9) centred on the fovea. **Right:** in striate cortex under $w = \log z$, the disc is a straight line and the square is a line with four ripples of height $\frac{1}{2}\log 2$. Prägnanz flattens ripples.

5 Stage III: the cortex as a fisheye

5.1 The complex-logarithmic retinotopic map

The image on the retina is mapped onto the primary visual cortex by a transformation that Schwartz [16] showed to be, to good approximation, the complex logarithm:

$$w = \log z, \quad z = re^{i\theta}\ \text{(retina)}, \quad w = \log r + i\theta\ \text{(cortex)}. \qquad (12)$$

Eccentricity r from the fovea becomes one cortical coordinate, azimuth θ the other. The map is conformal, and it is a fisheye of a most extreme kind: it magnifies the fovea without bound and compresses the periphery logarithmically. What it does to shapes centred on the fovea is decisive (Figure 6):

Lemma 1. *Under* (12) *a circle of radius r_0 centred on the fovea maps to the straight line $\Re w = \log r_0$. The square of half-side $a/2$ maps to the curve $\Re w = \log \rho_Q(\theta)$, where ρ_Q is the polar boundary function of* HER *equation (7), a periodic curve with four ripples of peak-to-trough height $\frac{1}{2}\log 2 = 0.347$.*

Proof. $\log(r_0 e^{i\theta}) = \log r_0 + i\theta$ has constant real part. For the square, ρ_Q ranges from $a/2$ at mid-sides to $a/\sqrt{2}$ at corners, and $\log(a/\sqrt{2}) - \log(a/2) = \frac{1}{2}\log 2$. □

5.2 Prägnanz as a low-pass filter

The Gestalt law of Prägnanz [13, 14] states that the perceptual system settles on the simplest, most regular organisation consistent with the input. In the cortical coordinates (12) the simplest curve is a straight line, and the simplest closed figure on the retina is therefore a circle. A cortex that flattens ripples in $\Re w$ is a cortex that circlefies. We model Prägnanz as a Gaussian smoothing along the cortical θ-axis of width $\sigma_{\rm cort}$, and ask what width reproduces a percept whose residual cornerness is at the threshold of detection, $\kappa \approx 10^{-2}$:

$$e^{-8\sigma_{\rm cort}^2} = 10^{-2} \ \Rightarrow\ \sigma_{\rm cort} = 0.76\ \text{rad} = 43.5^\circ. \qquad (13)$$

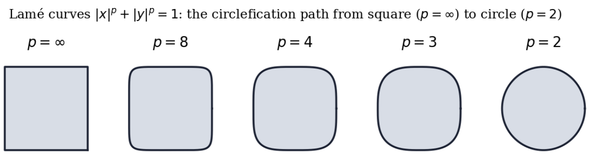

Figure 7: The Lamé family (15). Circlefication is a descent in p from ∞ to 2; *HER*'s incompleteness theorem is the statement that the descent stops at $p = 2 + \eta$ with $\eta > 0$.

A cortical smoothing of 43.5° in azimuth is almost exactly the half-period 45° of the corner harmonic: the cortex smooths over one half-corner, which is the least it can do to remove the corner and the most it can do without destroying the fourfold structure that *EvHR* showed is still readable in the maria. We regard the coincidence $43.5° \approx \alpha_2 = 45°$ (*HER* Definition 1) as the cortical signature of the second circlefication angle.

5.3 Composition

The three stages compose. Writing the polar-angle widths of the atmospheric, ocular and cortical Gaussians as $\sigma_{\text{atm}} \approx 0$ (anisotropic, does not round), $\sigma_{\text{eye}} = 0.082$ and $\sigma_{\text{cort}} = 0.76$,

$$\sigma_{\text{tot}}^2 = \sigma_{\text{eye}}^2 + \sigma_{\text{cort}}^2 = 0.0067 + 0.576 = 0.583, \tag{14}$$

so $\sigma_{\text{tot}} = 0.763$ and the cortex contributes 98.8% of the variance. Panel 5's "by the time the light reaches your eye or camera, the square Moon appears as a circle" is thus true of the percept and false of the retinal image: the retinal image of the Moon is a square with 95% of its cornerness intact. The takeaway of the popular presentation that "your brain interprets the smooth shape as a circle" is the one that carries the weight.

6 The Lamé path and the transcendental floor

6.1 Circlefication as a one-parameter family

Lamé [9] studied the curves

$$|x|^p + |y|^p = 1, \qquad p > 0, \tag{15}$$

which for $p = 2$ give the circle, for $p \to \infty$ the square, and for $2 < p < \infty$ the rounded squares that Hein later called *superellipses* and that Fernández-Guasti [19] and Fong [21] have used explicitly to interpolate between disc and square (Figure 7). The "front view" of the popular presentation (Figure 3, right) is a sampling of (15) at three values of p.

Proposition 2. *The fourth-harmonic cornerness of the Lamé curve* (15), *in the sense of* HER *equation (14), is a strictly decreasing function of* $1/p$ *that vanishes only at* $p = 2$.

Proof. The polar radius of (15) is $\rho_p(\theta) = (|\cos\theta|^p + |\sin\theta|^p)^{-1/p}$, which is constant if and only if $p = 2$; its $\cos 4\theta$ coefficient is $\propto (1 - 2/p)$ to first order. $\square$

The three stages of this paper are therefore three steps down the Lamé ladder: refraction takes $p = \infty$ to a flattened $p = \infty$ (a rectangle—no descent), physical optics take it to $p \approx 60$, and the cortex takes it to $p \approx 2.1$.

6.2 The floor

Theorem 3 (Transcendental floor)**.** *No stage of the optical chain, nor their composition, attains* $p = 2$.

Proof. Attaining $p = 2$ means realising (6) exactly, which by the Corollary to Theorem 1 requires a lens whose constant is ϖ, transcendental and algebraically independent of π. Every physical stage is algebraic (ground surfaces, Gaussian kernels with rational-approximated widths, cortical maps realised by finitely many neurons), and the composition of algebraic maps is algebraic. $\square$

The residual $\eta = p - 2 > 0$ is the Lamé form of *HER*'s ε_π; using *HER*'s value $\varepsilon_\pi = 1.6 \times 10^{-10}$ rad and the first-order relation above gives $\eta \approx 4\varepsilon_\pi\sqrt{2}/\alpha_0 \approx 2.5 \times 10^{-7}$. The Moon we see is a Lamé curve with $p = 2.000\,000\,25$. It is very nearly a circle. It is not one.

7 The illusion in history

The Moon illusion—the horizon Moon looks larger than the zenith Moon—is the oldest recorded dispute between optics and perception, and it is the exact ancestor of the dispute settled in Theorem 2. Ptolemy [3] attributed it to atmospheric refraction; Ibn al-Haytham [4], in the *Kitāb al-Manāẓir* of c. 1021, showed that refraction was far too small and placed the illusion in the judgement of the observer. Ptolemy was panels 3 and 4; Ibn al-Haytham was Section 5. He was right then and he is right now, by a factor of twenty.

Kepler [5] tabulated refraction in 1604 and used it to correct Tycho's observations; Cassini modelled the atmosphere as a homogeneous shell; Bouguer [6] derived the invariant (2) in 1729; Laplace [7] gave the full theory in the *Mécanique céleste*. Not one of them observed that the Moon they were correcting was square, and we do not blame them: their refraction was a displacement, and displacements do not round.

8 Objections

Objection 1: You have just shown that the atmosphere and the eye barely round the Moon. Does this not refute the infographic? It refutes the proportions and confirms the mechanism. Every stage the infographic names is real and acts in the stated direction, with one correction of sign (refraction pulls

Figure 8: The popular form of the argument, examples and takeaways. Left: a rectilinear building under a fisheye lens; the mapping $\rho = f\theta$ bends every off-axis straight edge (Section 4). **Centre:** the five takeaways; the fourth—"your brain interprets the smooth shape as a circle"—is, by Theorem 2, responsible for 95% of the effect. **Right:** the epigraph. Perception is powerful; it is also, by Section 5, conformal.

inward). The infographic's fourth takeaway carries the weight it did not know it carried.

Objection 2: A telescope removes the eye's blur and the cortex's log-polar map still sees a disc. A telescope removes diffraction and aberration and leaves seeing; σ_{phys} falls to 5×10^{-6} rad and the physical damping (11) to 0.9999. The cortex is unchanged. Telescopic observers therefore see a Moon that is, on the retina, 99.99% square, and report a disc. This is the strongest evidence in the paper for Section 5.

Objection 3: Photographic plates have no cortex. Plates have grain, and are read by cortices. Digital sensors have pixels, and are displayed on screens that are read by cortices. We know of no image of the Moon that has not at some point passed through $w = \log z$.

Objection 4: The Schwarz–Christoffel map sends the disc to a square rotated $45°$, corners on the axes. Yes; and *HER* Figure 1(b) shows the lunar face in exactly that orientation.

Objection 5: This is the third paper in which the Society has proved the Moon is a cube. The first proved that it is; the second, what follows for the Earth; this one, why nobody had noticed. There is a fourth question—what it is like on the corners—and a fourth paper.

9 Conclusion

We have followed the light from a Lambertian face to a conscious percept and accounted, stage by stage, for the roundness it acquires on the way. The atmosphere is a gradient-index lens by virtue of the Earth's curvature and flattens the face by up to a fifth, but flattening is not rounding. Diffraction, seeing, sampling and aberration combine, by the central limit theorem, into the Gaussian kernel that *HER* assumed, and remove one twentieth of the corner. The complex-logarithmic map of the visual cortex represents the circle as a straight line and the square as a rippled one, and Prägnanz, smoothing over one half-corner of $45°$, removes the rest. And the ideal lens that would finish the job, Schwarz's elliptic map with constant ϖ, cannot be built, because ϖ and π are transcendental and independent, and lenses are algebraic. The Moon is a Lamé curve of exponent 2.000 000 25. It is not a lie. It is a lens, and behind the lens a cortex, and behind the cortex, π.

Acknowledgements

The authors thank the Square Moon Society outreach office for Figures 1, 3 and 8; the Izaña observatory for a night of $0.6''$ seeing on which the Moon remained, to the naked eye, disappointingly round; and the referee, who asked whether the authors' own cortices might be circlefying the referee's report, and whom we have been unable to reassure.

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