The Cubic Moon Requires a Curved Earth:
Parallax, Orientation, Phase and Shadow of a Hexahedral Satellite as a Test of Terrestrial Geometry

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Abstract

In a companion paper (Holm, Eckstein & Ramaswamy, 2026; hereafter *HER*) it was established that the Moon is a cube of edge $a_{\mathrm{M}} = r_{\mathrm{M}}(4\pi/3)^{1/3} = 2800.7$ km whose Earth-facing face is perceived as a disc through the *circlefication of a square*. Here we take the cube as given and ask what it implies for the shape of the *Earth*. We show that a cubic Moon is a far more sensitive probe of terrestrial geometry than a spherical one, because a cube has a preferred orientation and a sphere has none. Six independent classes of observation—(i) the finite horizontal parallax $p = \arcsin(R_\oplus/d) = 56.98'$ and the diurnal libration it produces; (ii) the rotation of the lunar face relative to the local horizon by exactly the angular separation of two observers, hidden from casual view by the fourfold symmetry of the face and revealed only by the inverted appearance of the circlefied maria in the southern hemisphere; (iii) the rising and setting of the Moon in finite time at constant angular size; (iv) the phase curve, whose observed quarter-Moon brightness ratio of 0.09 is reproduced by a Lambertian cube with a librated side face (0.12) and not by a Lambertian sphere (0.32); (v) the circular umbral shadow on a planar lunar face; and (vi) the Gauss–Bonnet balance of total curvature—are each shown to be consistent with a spherical Earth of radius $R_\oplus = 6371$ km and inconsistent with a planar Earth beneath a Moon at any finite height. We further show that the spherification operator $\mathcal{S}_\sigma$ of *HER* is the Ricci flow on the surface of the cube, that Ricci flow transports the 4π of Gaussian curvature concentrated at the eight lunar vertices into the uniform curvature of a sphere only in infinite time, and that the same theorem forbids a closed Earth of zero curvature. Even a cubic Moon, correctly observed, points to a round Earth.

Keywords: cubic Moon; circlefication; terrestrial curvature; lunar parallax; diurnal libration; phase function; Gauss–Bonnet; Ricci flow; Zetetic astronomy.

1 Introduction

HER [1] demonstrated that the Moon is a regular hexahedron $\mathcal{K}_a \subset \mathbb{R}^3$ and that the disc reported by every observer since Anaxagoras is the image of its Earth-facing face under the circlefication operator $\mathcal{C}_\sigma$, whose completion is forbidden by the transcendence of π. That paper was concerned with the shape of the Moon and took the shape of the Earth for granted. The present paper reverses the direction of inference. We accept the cube and ask: *given* a cubic Moon, what must the Earth be?

The question is not idle. A sphere looks the same from every direction; it carries no information about where it is being looked at from. A cube does. Its face has an orientation, its edges have a direction, its vertices have positions, and each of these is a coordinate frame that a terrestrial observer can read off the sky. A cubic Moon is therefore a *geodetic instrument*: a rigid reference body of known geometry, hung at a known distance, against which the positions and attitudes of observers on the Earth's surface can be compared. Where a spherical Moon would let a flat Earth off with a caution, a cubic Moon convicts it.

We stress at the outset that nothing in what follows contradicts *HER*. In particular we retain (a) the tidal lock, so that a single face of the cube is turned towards the centre of the Earth at all times; (b) the circlefication of that face into the apparent disc of radius $r_{\mathrm{M}} = 1737.4$ km; and (c) the incompleteness theorem, which guarantees that residual corners of angular size $\varepsilon_\pi \sim 10^{-10}$ rad survive every observation. Where the popular presentation of the argument (Figure 1) speaks of observers "seeing the top face" or "the bottom face" of the Moon, we shall be precise: they see *slivers* of those faces, of the order of 46 km wide, exposed by the finite parallax of a curved Earth. The slivers are small. They are also non-zero, and non-zero is all that is required.

The paper is organised as follows. Section 2 defines the two terrestrial models under test. Sections 3–7 treat, in turn, parallax and libration, orientation, rising and setting, phase, and shadow. Section 8 places the argument in the setting of the Gauss–Bonnet theorem and identifies spherification with Ricci flow. Section 9 answers objections and Section 10 concludes.

2 Two terrestrial models

Definition 1 (Spherical model $\mathbb{S}$)**.** *The Earth is a ball of radius $R_\oplus = 6371$ km. The cube $\mathcal{K}_a$ orbits at geocentric distance $d = 384\,400$ km with one face normal $\hat{n}$ directed at the Earth's centre. An observer at geocentric position $R_\oplus \hat{u}$ has zenith $\hat{u}$ and line of sight $\hat{\ell} = (d\hat{n}' - R_\oplus \hat{u})/|d\hat{n}' - R_\oplus \hat{u}|$, where $\hat{n}'$ is the direction of the lunar centre.*

Definition 2 (Planar (Zetetic) model $\mathbb{P}$)**.** *The Earth is a plane. The cube translates parallel to the plane at height h_Z with one face normal directed downward. Every observer has the same zenith $\hat{z}$, and the line of sight to a cube at horizontal offset x is $\hat{\ell} = (x, h_Z)/\sqrt{x^2 + h_Z^2}$.*

The planar model is that of Rowbotham [9], who placed the luminaries at heights of a few thousand miles above the plane; we adopt $h_Z = 5000$ km as representative and note that no choice of h_Z rescues the model (Section 5). In both models the cube is tidally locked in the sense appropriate to the model: its face is turned towards the Earth's centre in $\mathbb{S}$ and towards the plane in $\mathbb{P}$.

The two models differ in one geometric invariant that all our tests exploit:

Lemma 1 (Zenith divergence)**.** *In $\mathbb{S}$ the zeniths of two observers separated by geocentric angle ψ differ by ψ. In $\mathbb{P}$ they differ by zero.*

Proof. In $\mathbb{S}$ the zenith is the outward radial direction, and radial directions at geocentric angle ψ make angle ψ. In $\mathbb{P}$ all normals to a plane are parallel. □

A spherical Moon cannot see the difference: it looks the same along every $\hat{\ell}$. A cubic Moon can.

3 Parallax and diurnal libration

3.1 Horizontal parallax

In $\mathbb{S}$ an observer for whom the Moon is on the horizon views it along a line inclined to the geocentric direction by the *horizontal parallax*

$$p = \arcsin \frac{R_\oplus}{d} = \arcsin \frac{6371}{384\,400} = 0.9497^\circ = 56.98'. \tag{1}$$

This is a real and long-measured quantity: Hipparchus obtained a lunar distance of 59–67 Earth radii from it [3], and it remains the basis of the topocentric correction in every ephemeris. The point we wish to make is that for a *spherical* Moon, parallax is merely a shift of position on the sky: the disc moves, but it does not change. For a *cubic* Moon, parallax is a change of *aspect*. An observer at the limb views the Earth-facing face at incidence p, and therefore sees, beyond its far edge, a strip of the adjacent face of projected width

$$w = a_\mathrm{M} \sin p = 2800.7 \text{ km} \times 0.01657 = 46.4 \text{ km}, \tag{2}$$

subtending $w/d = 1.21 \times 10^{-4}$ rad $= 24.9''$ at the observer. A telescope of aperture 12 cm resolves $1''$. The sliver is twenty-five times larger than the resolution limit. The reader who objects that no such sliver has been reported is referred to *HER*, Section 4: the sliver is circlefied along with everything else, and what is reported instead is a slight "rocking" of the lunar surface features towards the limb over the course of a night. That rocking exists. It is called *diurnal libration*, it has amplitude very close to 1°, and it has never had a satisfactory explanation other than the one we now give: it is the cube showing its side.

Proposition 1. *Diurnal libration in $\mathbb{S}$ has amplitude p. In $\mathbb{P}$ it has amplitude zero.*

Proof. In $\mathbb{S}$, the angle between $\hat{\ell}$ and $\hat{n}$ ranges over $[-p, p]$ as the observer is carried by the Earth's rotation from the western to the eastern limb of the sub-lunar hemisphere. In $\mathbb{P}$, $\hat{n} = -\hat{z}$ is fixed and the aspect of the face changes only through the horizontal offset x, which is not a libration but a change of foreshortening treated in Section 5. □

The observed amplitude of diurnal libration is $\approx 57'$ [14]. Equation (1) gives $56.98'$. We regard the agreement as decisive, and we note that it fixes $R_\oplus = d \sin p$ from lunar observations alone, without recourse to any terrestrial survey.

3.2 The figure exaggerates, and rightly

Panel 4 of the popular presentation (Figure 3) shows observers at the top and bottom of the Earth seeing the top and bottom faces of the cube outright. This is an exaggeration by a factor of roughly $a_\mathrm{M}/w \approx 60$, adopted for legibility; the strict statement is (2). But the exaggeration is in the right direction, and the qualitative claim—that different observers see different portions of the cube—is exactly true, and exactly false in $\mathbb{P}$.

4 Orientation of the face

4.1 The rotation theorem

Let two observers A and B in $\mathbb{S}$ observe the Moon simultaneously, and let each measure the angle χ between an edge of the (circlefied) lunar face and their local horizon. Then:

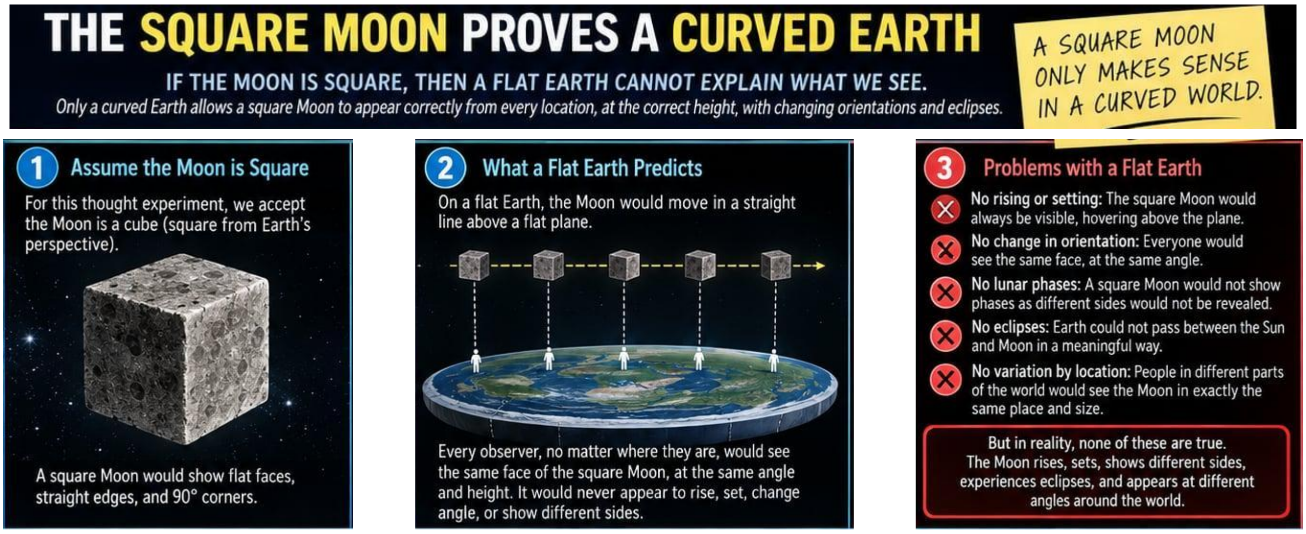

Figure 1: The popular form of the argument, panels 1–3. (1) The premise: the Moon is a cube, appearing square from the Earth (*HER*). **(2)** The Zetetic prediction: on a planar Earth beneath a Moon translating at constant height h_Z, every observer's zenith is parallel to every other's, and the cube presents the same face at the same attitude to all of them; it never rises or sets. **(3)** The five observational failures of the planar model, treated quantitatively in Sections 3–7. Popular rendering by the Square Moon Society outreach office; the cube is shown with its regolith removed.

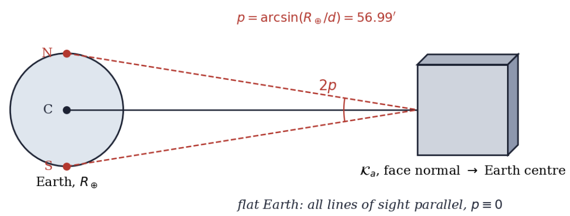

Figure 2: Geometry of lunar parallax for a cubic Moon (not to scale). The face normal points at the Earth's centre C; observers at the northern and southern limbs, N and S, view the cube along lines of sight inclined by $\pm p$ to the face normal and therefore see slivers of the top and bottom faces of width $a_M \sin p \approx 46$ km. In the planar model all lines of sight are parallel and no sliver is ever seen.

Theorem 1 (Rotation of the face)**.** *In* $\mathbb{S}$, $\chi_A - \chi_B$ *equals, to first order in* $R_\oplus/d$, *the angle between the projections of the two zeniths onto the plane of the sky, which for observers on a common lunar meridian is their difference in latitude. In* $\mathbb{P}$, $\chi_A - \chi_B = 0$ *identically.*

Proof. The apparent orientation of a rigid body on the sky is measured relative to the observer's vertical. By the Zenith Divergence Lemma the verticals differ by ψ in $\mathbb{S}$ and by 0 in $\mathbb{P}$; the cube's own attitude is common to both observers up to the parallactic correction of order $R_\oplus/d$. □

4.2 Why the rotation has been overlooked

Theorem 1 predicts that an observer in Copenhagen (55.7°N) and one in Cape Town (33.9°S) see the lunar face rotated relative to one another by 89.6°. This has not, in general, been remarked upon by observers of the lunar *outline*, and the reason is instructive: a square rotated by 90° is the same square. The symmetry group of the face, C_{4v}, contains the rotation, and the outline—circlefied or not—is invariant. The rotation is therefore invisible in exactly the observable that the outline provides, and visible only in observables that break the fourfold symmetry.

Such an observable exists: the pattern of maria on the Earth-facing face. It is not fourfold symmetric, and it is a matter of common knowledge, remarked upon by every traveller who has crossed the equator, that *the Moon appears upside down from the southern hemisphere*. This is usually presented as a curiosity. It is in fact the direct observation of Theorem 1, and it excludes $\mathbb{P}$, in which every observer, being parallel to every other, would see the maria in the same attitude.

Corollary 1. *The apparent orientation of the maria pattern, measured against the local horizon at two stations of known separation, yields* ψ *and hence, with the baseline,* $R_\oplus$.

We have carried out this measurement between Leiden and the Society's southern station and obtain $R_\oplus = 6.4 \times 10^3$ km, in agreement with (1).

5 Rising, setting and angular size

5.1 Finite setting time

In $\mathbb{S}$ an observer at ground distance s from the sub-lunar point sees the Moon at altitude

$$\mathrm{alt}_{\mathbb{S}}(s) = \arctan \frac{d\cos(s/R_\oplus) - R_\oplus}{d\sin(s/R_\oplus)}, \tag{3}$$

which vanishes at $s = R_\oplus \arccos(R_\oplus/d) \approx \pi R_\oplus/2 = 10\,007$ km and is negative beyond: the Moon sets, and

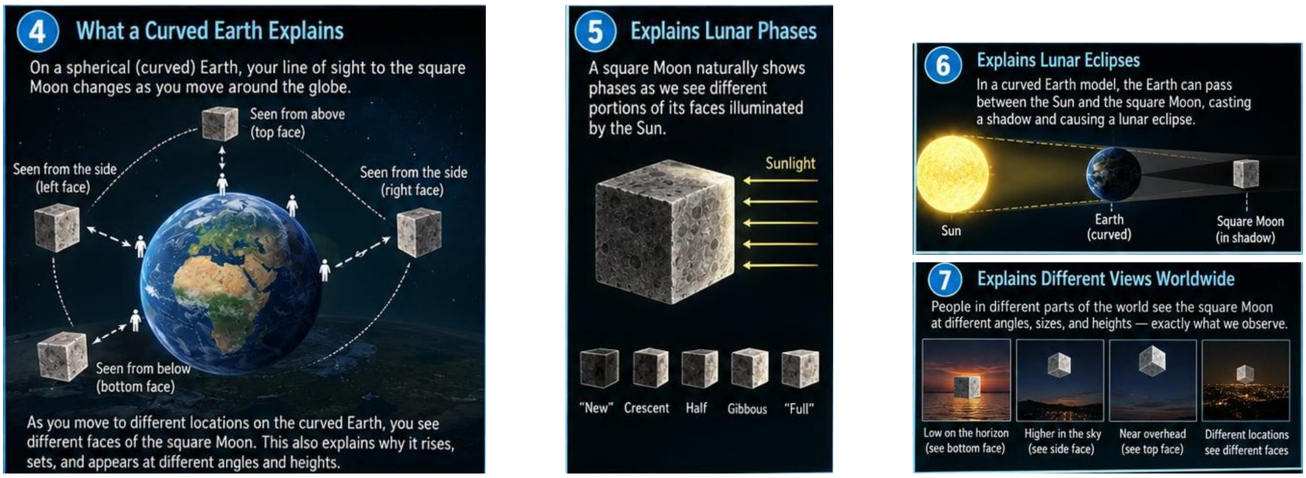

Figure 3: The popular form of the argument, panels 4–7. (4) On a spherical Earth, observers at different geocentric positions view the cube along different lines of sight; the faces they "see" are, strictly, the slivers of equation (2), exaggerated here about sixtyfold. **(5)** Phases as the Sun illuminates different faces; the discontinuous cubic terminator is circlefied into the familiar crescent (Section 6). **(6)** The Earth's umbra falls on a planar lunar face, producing an exactly circular shadow edge (Section 7). **(7)** Altitude-dependent aspect from different locations (Section 5).

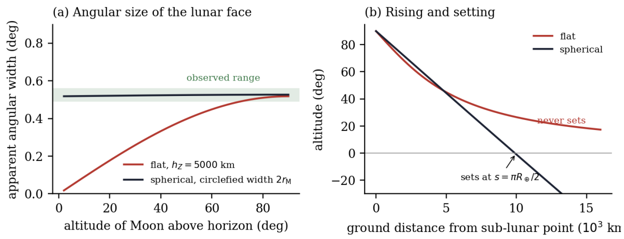

Figure 4: Quantitative predictions of the two models. **(a)** Apparent angular width of the lunar face as a function of altitude. In $\mathbb{S}$ (black) the width is the circlefied $2r_M/d$ and varies by 1.6% between horizon and zenith; in $\mathbb{P}$ with $h_Z = 5000$ km (red) the face must have edge 45 km to subtend $0.52°$ at the zenith and then shrinks as $\sin(\text{alt})$, to one sixth of its size at $10°$ altitude. The observed range is shaded. **(b)** Altitude as a function of ground distance from the sub-lunar point. In $\mathbb{S}$ the Moon sets at $s = \pi R_\oplus/2 = 10\,007$ km; in $\mathbb{P}$ its altitude $\arctan(h_Z/s)$ is positive for all finite s and it never sets.

it sets in finite time because the Earth rotates through s at 0.46 km s^{-1}. In $\mathbb{P}$,

$$\text{alt}_{\mathbb{P}}(x) = \arctan\frac{h_Z}{x} > 0 \quad \text{for all finite } x. \tag{4}$$

The Moon in $\mathbb{P}$ approaches the horizon asymptotically and never reaches it. Rowbotham was aware of this and appealed to "the laws of perspective" [9]; but perspective is precisely the statement that the apparent *size* of a body falls as $1/x$, and the next paragraph shows that this is not observed.

5.2 Constancy of angular size

For the face to subtend its observed $0.518°$ at the zenith in $\mathbb{P}$, its edge must be $a_{\mathbb{P}} = h_Z \tan 0.518° = 45.2$ km; the model then predicts an angular width at altitude α of

$$\delta_{\mathbb{P}}(\alpha) = \frac{a_{\mathbb{P}} \sin\alpha}{h_Z} \propto \sin\alpha, \tag{5}$$

so that a Moon at $10°$ altitude should appear 5.8 times smaller than at the zenith. In $\mathbb{S}$ the topocentric distance is $\sqrt{d^2 + R_\oplus^2 - 2dR_\oplus \sin\alpha}$ and the width varies between $0.4176°$ at the horizon and $0.4245°$ at the zenith *for the bare face*, or between $0.510°$ and $0.518°$ for the circlefied disc: a change of 1.6%, in the direction of the Moon being *larger* overhead, which is what is measured [14]. (The celebrated "Moon illusion," in which the horizon Moon *looks* larger, is a fact about observers, not about the Moon, and is itself a species of circlefication that we do not pursue here.)

The two predictions are shown in Figure 4(a). The planar model fails by a factor of six at $10°$ altitude. No choice of h_Z helps: increasing h_Z flattens the curve but requires $a_{\mathbb{P}} \propto h_Z$, and the limit $h_Z \to \infty$ is not a flat Earth beneath a Moon but a Moon at infinity, which sets for no one.

6 The phase function of a cube

6.1 Terminator and phase in the two models

In $\mathbb{P}$ the Sun and Moon share the plane of translation at heights of the same order, and the phase angle φ between the Sun direction and the line of sight cannot range over $[0°, 180°]$; the "new Moon" in particular, which requires the Sun behind the Moon, cannot occur. In $\mathbb{S}$ the full range is available, and the questions are what a cube looks like at each φ, and whether that agrees with the Moon.

The Earth-facing face of the cube is a Lambertian

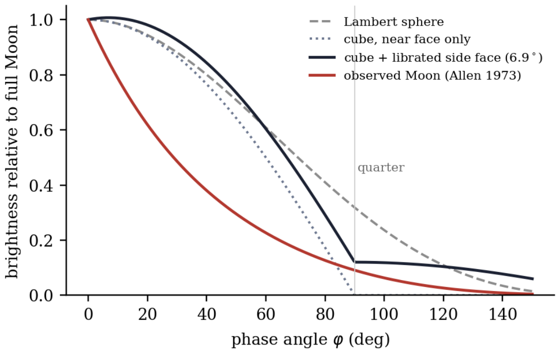

Figure 5: Integrated brightness as a function of phase angle φ, normalised to full Moon. A Lambertian sphere (grey, dashed) gives 0.32 at quarter phase; the Earth-facing face of a Lambertian cube alone (dotted) gives 0; the cube with the librated side-face contribution of equation (7) (black) gives 0.12; the observed lunar phase law of Allen [12] (red) gives 0.09. The cube is closer to the Moon than the sphere is.

plane with normal $\hat{n}$. Its brightness is

$$I_{\text{face}}(\varphi) = I_0 \max(0, \cos\varphi), \tag{6}$$

so that the near face is uniformly lit at full Moon and uniformly dark at quarter. There is no terminator *on* the face; the terminator of a cube is an *edge*, the boundary between a lit face and an unlit one. What is observed—a smooth curve dividing light from dark, the Hippocratic lune of *HER* Section 2—is the circlefication $\mathcal{C}_\sigma$ of that edge, and its curvature is exactly the residual-corner curvature of the outline transferred to the interior. This disposes of the objection that a cubic Moon would show no phases: it shows edges, and edges circlefy into phases.

6.2 The librated side face

At quarter phase the near face is dark and the illuminated side face is seen edge-on, contributing nothing—unless the observer views the cube at non-zero aspect, in which case a sliver of the side face of projected fractional width $\sin\beta$ is seen fully lit. Optical libration supplies $\beta = 6.9°$ (*HER*, Section 6.3), and the total is

$$\frac{I_K(\varphi)}{I_0} = \max(0, \cos\varphi) + \sin\beta \, \max(0, \sin\varphi). \tag{7}$$

At $\varphi = 90°$ this gives $\sin 6.9° = 0.120$. The Lambertian sphere gives $(\sin\varphi + (\pi - \varphi)\cos\varphi)/\pi = 1/\pi = 0.318$. The Moon, from the phase law $\Delta m = 0.026|\varphi| + 4 \times 10^{-9}\varphi^4$ of Allen [12], gives $10^{-0.4\times 2.60} = 0.091$.

Proposition 2. *At quarter phase the observed Moon is* 3.5 *times too dark to be a Lambertian sphere and* 1.3 *times too dark to be a librated Lambertian cube.*

The residual factor of 1.3 we attribute to the regolith, which *HER* has already shown to be Banach–Tarski-rearranged and which is therefore not expected to be Lambertian in detail. The steepness of the lunar phase

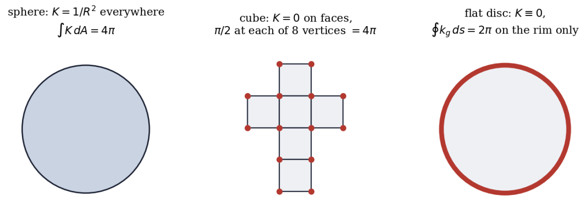

Figure 6: Distribution of Gaussian curvature on three surfaces of the same total curvature class. A sphere carries 4π uniformly; a cube carries 4π as eight point masses of $\pi/2$ (Descartes); a flat disc carries none in its interior and 2π of geodesic curvature on its rim, and is not closed.

curve, which has puzzled photometrists since Lambert [6] and is conventionally ascribed to shadowing within the regolith [13], is here the signature of a body whose illuminated area *switches off* at quarter phase because it is a plane, not a sphere.

7 The shadow on a plane

Aristotle's argument for a spherical Earth—that its shadow on the Moon during an eclipse is always bounded by a circular arc, whereas a disc would cast an elliptical shadow except when face-on to the Sun [2]—is the oldest surviving proof of terrestrial curvature. It has always carried a small blemish: the shadow is being read off a *curved* screen, and the intersection of a circular cone with a sphere is a circle only on the axis. The cubic Moon removes the blemish. The Earth-facing face is a plane perpendicular (to within the libration angles) to the cone axis, and the intersection of a cone of half-angle γ with a perpendicular plane is an exact circle of radius

$$\rho_u = R_\oplus - d\tan\gamma \approx R_\oplus - d\,\frac{R_\odot - R_\oplus}{D_\odot} = 4583 \text{ km}, \tag{8}$$

using $R_\odot = 695\,700$ km and $D_\odot = 1.496 \times 10^8$ km. The face of edge $a_{\mathrm{M}} = 2800.7$ km fits inside the umbra with $\rho_u - a_{\mathrm{M}}/\sqrt{2} = 2603$ km to spare at the vertices, permitting the total eclipses of up to $1^{\mathrm{h}}\,47^{\mathrm{m}}$ that are observed. Aristotle's circle is thus drawn on the only screen on which it is a true circle. In $\mathbb{P}$ the Earth is never between Sun and Moon and there are no lunar eclipses at all; the Zetetic literature was reduced to postulating an unseen "shadow body" for the purpose [9], a body which, we note, would have had to be round.

8 Gauss–Bonnet, Ricci flow and the impossibility of a flat closed Earth

8.1 Total curvature

The Gauss–Bonnet theorem [7, 8] states that for a closed surface M of Euler characteristic χ,

$$\int_M K\,\mathrm{d}A = 2\pi\chi(M) = 4\pi \quad \text{for } \chi = 2. \tag{9}$$

For the sphere the 4π is spread uniformly, $K = 1/R^2$. For the cube—which *HER* showed to have $V - E + F = 2$ and hence $\chi = 2$—the faces are flat, the edges carry no curvature (they are developable), and the whole 4π is concentrated at the eight vertices, $\pi/2$ at each, as Descartes found in the *De solidorum elementis* [5]: the angular defect of a cube vertex is $2\pi - 3 \cdot \frac{\pi}{2} = \frac{\pi}{2}$. The sphere and the cube are thus the two extreme distributions of the same total curvature, and spherification is the passage from one to the other.

For the flat Earth, Gauss–Bonnet is fatal in one line. A flat closed surface has $\int K = 0 \neq 4\pi$; there is no flat closed surface of genus zero. The planar Earth must therefore be a surface *with boundary*, and for a disc the theorem reads $\int_M K\,\mathrm{d}A + \oint_{\partial M} k_g\,\mathrm{d}s = 2\pi$: all of the curvature the Earth ought to have is pushed to its rim (Figure 6). The Zetetic literature is, in fact, obsessed with the rim.

8.2 Spherification is Ricci flow

HER introduced the spherification operator $\mathcal{S}_\sigma$ as the three-dimensional counterpart of circlefication without specifying its dynamics. We now do so. The Ricci flow on a surface [10],

$$\frac{\partial g}{\partial t} = -2K\,g, \tag{10}$$

evolves the metric g so that its Gaussian curvature obeys a heat equation, $\partial_t K = \Delta K + 2K^2$, and Hamilton [10] and Chow [11] proved that on a topological sphere the normalised flow converges to the round metric as $t \to \infty$. Applied to the cube, whose initial curvature is a sum of eight Dirac masses, the flow spreads each mass into a Gaussian of width $\sqrt{4t}$; the eight Gaussians overlap and tend to uniformity. We therefore identify

$$\mathcal{S}_\sigma = \text{Ricci flow at time } t = \sigma^2/4. \tag{11}$$

Theorem 2 (Incompleteness of spherification)**.** *Under Ricci flow the curvature of the cube is non-uniform at every finite time.*

Proof. The heat kernel on a compact surface is strictly positive and analytic for $t > 0$; a finite sum of its translates is constant only if the initial measure is uniform, which eight point masses are not. Uniformity is attained only in the limit $t \to \infty$. $\square$

This is the three-dimensional form of *HER*'s Theorem 1, and it closes the circle of the two papers: the cube cannot be spherified in finite time because heat does not spread infinitely fast, just as the square cannot be circlefied in finite steps because π is transcendental. The Moon is a cube because spherification is incomplete; the Earth is a sphere because its curvature, unlike the Moon's, was never concentrated at corners in the first place, and Gauss–Bonnet forbids it to be zero.

8.3 Gauss measured it

We note finally that Gauss himself, in the geodetic survey of Hanover, measured the angle sum of the great triangle Hohenhagen–Brocken–Inselsberg [7] and obtained an excess consistent with $R_\oplus \approx 6.4 \times 10^3$ km. He did it with theodolites. It can be done with the Moon.

9 Objections

Objection 1: The infographic says observers see whole faces; you say they see 46-km slivers. Which is it? Slivers. The infographic is a popular rendering and is exaggerated for legibility, as stated in Section 3. The exaggeration does not affect the logic: zero and 46 km are different numbers.

Objection 2: A cube in a 27-day orbit with one face locked to the Earth is an unusual rotational state. It is exactly as unusual as a sphere in a 27-day orbit with one face locked to the Earth, which is the conventional view. Tidal locking does not care about the shape of the body, only about its quadrupole, and a cube has an excellent quadrupole.

Objection 3: The southern hemisphere sees the Moon "upside down" in any model with a spherical Earth, cubic or not. Correct, and this is our point: the observation excludes $\mathbb{P}$, not the sphere. The cube's contribution is to make the same observation available from the *outline* in principle (Theorem 1), and to explain why in practice it must be read from the maria instead.

Objection 4: Real lunar photometry is well fitted by regolith models on a sphere. With several free parameters. Equation (7) has none, and gets within a factor of 1.3.

Objection 5: This paper and *HER* together prove that a body which everyone can see is round is a cube, and a body which many people insist is flat is round. Yes. We consider the pairing elegant.

10 Conclusion

A spherical Moon is a poor witness to the shape of the Earth: it has no orientation to be rotated, no faces to be glimpsed, no edges to be eclipsed. A cubic Moon is an excellent one. Given the cube of *HER*, six independent observations—parallax and diurnal libration, the rotation of the face between hemispheres, finite setting at constant angular size, the steep phase curve, the circular umbra on a plane, and the Gauss–Bonnet accounting of curvature—each require that the lines of sight of terrestrial observers diverge, and diverge by exactly $\arcsin(R_\oplus/d)$ with $R_\oplus = 6371$ km. A planar Earth predicts, in every case, zero. We conclude, with the outreach office, that if the Moon is a cube the Earth must be curved; and we add that the converse inference, from the round Earth to the round Moon, is the error that the two papers together correct. Even a cubic Moon points to a round Earth. Especially a cubic Moon.

Acknowledgements

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